Imaging and nulling properties of sparse-aperture Fizeau interferometers

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General layout of a Fizeau interferometer

- All telescopes assumed to be identical
- All exit pupils optically conjugated with entrance pupils
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Coordinates systems

• Input pupils defined by vectors $\mathbf{OP}_n$
• Output pupils defined by vectors $\mathbf{O'P'}_n$

$1 \leq n \leq N$
Image of a sky object projected onto the sky

Most general Object-Image relationship:

\[
I(s) = \int \int_{s_0 \in \Omega_o} O(s_0) \cdot \text{PSF}_T(s-s_0) \left| \sum_{n=1}^{N} a_n \exp[i\phi_n] \exp[i k \xi(s_0,s)] \right|^2 \, d\Omega_o
\]

with

\[
\xi(s_0,s) = s_0 \cdot \text{OP}_n - s \cdot \text{O'} \cdot \text{P'}_n / m
\]

- \( \text{PSF}_T(s) \): PSF of an \textit{individual} collecting telescope, projected back onto the sky
- \( a_n \): amplitude transmission factor of the \( n^{th} \) telescope
- \( \phi_n \): phase-shift along the \( n^{th} \) interferometer arm for cophasing or nulling purpose
- \( k = 2\pi/\lambda \): wavenumber of \textit{monochromatic} electro-magnetic field
- \( m \): optical compression factor between telescopes and their relay optics
Golden rule of Fizeau interferometers

- First-order approximation
- Necessary condition: $O'P'_n = m \cdot OP_n$
- Object-Image relationship

$$I(s) = [\text{PSF}_T(s) \ F(s)] \ast O(s)$$

- Invariant PSF over the Field of view (FoV)

$$F(s) = \text{PSF}_T(s) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[i \ k \ s \ OP_n] \right|^2$$
Examples of application

- Effect of pupil aberrations
- Deviations from the “golden rule”: non-homothetic exit pupil combiners
- Simulation of images in nulling mode
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Effect of pupil aberrations

- Develop all vectors at 2\textsuperscript{nd}-order, e.g.

$$s = \begin{cases} \sin u \approx u \\ \cos u \sin v \approx v \\ \cos u \cos v \approx 1 - u^2/2 - v^2/2 \end{cases}$$

- Insert into the OPD:

$$\xi(s_0,s) \approx dz_n - dz_n'/m + u_0x_n + v_0y_n - (u'x_n + v'y_n)(l + dz_n'/F')/m$$

$$- dz_n(u_0^2 + v_0^2)/2 - dz_n'(u^2 + v^2)/2m$$

- Use general Object-Image relation:

$$I(s) = \int \int \int_{s_0 \in \Omega_0} O(s_0) \text{PSF}_I(s-s_0) \sum_{n=1}^{N} a_n \exp[i\phi_n] \exp[ik\xi(s_0,s)]^2 \, d\Omega_0$$

- No longer a convolution operator!
Pupil aberration: 2 telescopes

Achievable FoV: 1.5 arcsec

PSF at FoV centre:
- Invariant PSF

PSF at 0.3 arcsec:
- Curved fringes, changing PSF

PSF at 0.6 arcsec:
- Curved fringes, changing PSF

No pupil aberration

Aberrated pupil Case 1

Aberrated pupil Case 2
Pupil aberration: 8 telescopes

Achievable FoV

PSF at
FoV centre

PSF at
0.3 arcsec

PSF at
0.6 arcsec

No pupil
aberration

Aberrated pupil
Case 1

Aberrated pupil
Case 2

Invariant PSF
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Different combining schemes

From collecting telescopes

APS 1

Metrology beam 1

Fringe tracker

APS 2

Metrology beam 2

Axial combiner

(\(P'\))

O'

F'

Z

Focal plane

From collecting telescopes

B

Metrology beams 1-4

Crossed-cubes nuller

B'

(\(P'\))

O'

F'

Z

Focal plane
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**Axially Combined Interferometer (ACI)**

- Co-axial combination by means of a balanced set of beamsplitters
- Extinguishes all diffracted light originating from central star → Only leaves planet photons
- Specific Object-Image relationship

\[
I(s) = PSF_T(s) \times \left\{ \sum_{n=1}^{N} a_n \exp[i\phi_n] \exp[iksOP_n] \right\}^2 O(s)
\]
Crossed-cubes nuller (CCN)

- See the “Cheapest nuller in the World”:
  - One talk on Thursday afternoon, 2 posters on Wednesday afternoon
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Pseudo-images of a companion

Fake sky objects

Telescope array

Phase-shifts maps

Axial combiner

Maximal densification

Fizeau interferometer
Conclusion

• A simple Fourier optics formalism allows fast evaluation of nulling Fizeau interferometers performance
  – Including PSF, achievable Field of view (FoV), nulling maps and pseudo-images
  – Whatever is the telescope number N
  – In presence of optical defects (pupil aberrations)
  – Also applicable to non-Fizeau interferometers

• Simple formalism, no actual Fourier or Fresnel transforms required (gain in accuracy and computing time)

• Simulations show that the most efficient nulling schemes should use axial combiners or multi-axial combiners with maximal densification (e.g. Crossed-cubes nuller)
Previous publications

- “Cheapest nuller in the world: crossed beamsplitter cubes,” Proceedings of the SPIE vol. 9146, this conference

Perhaps one synthesis paper some day…