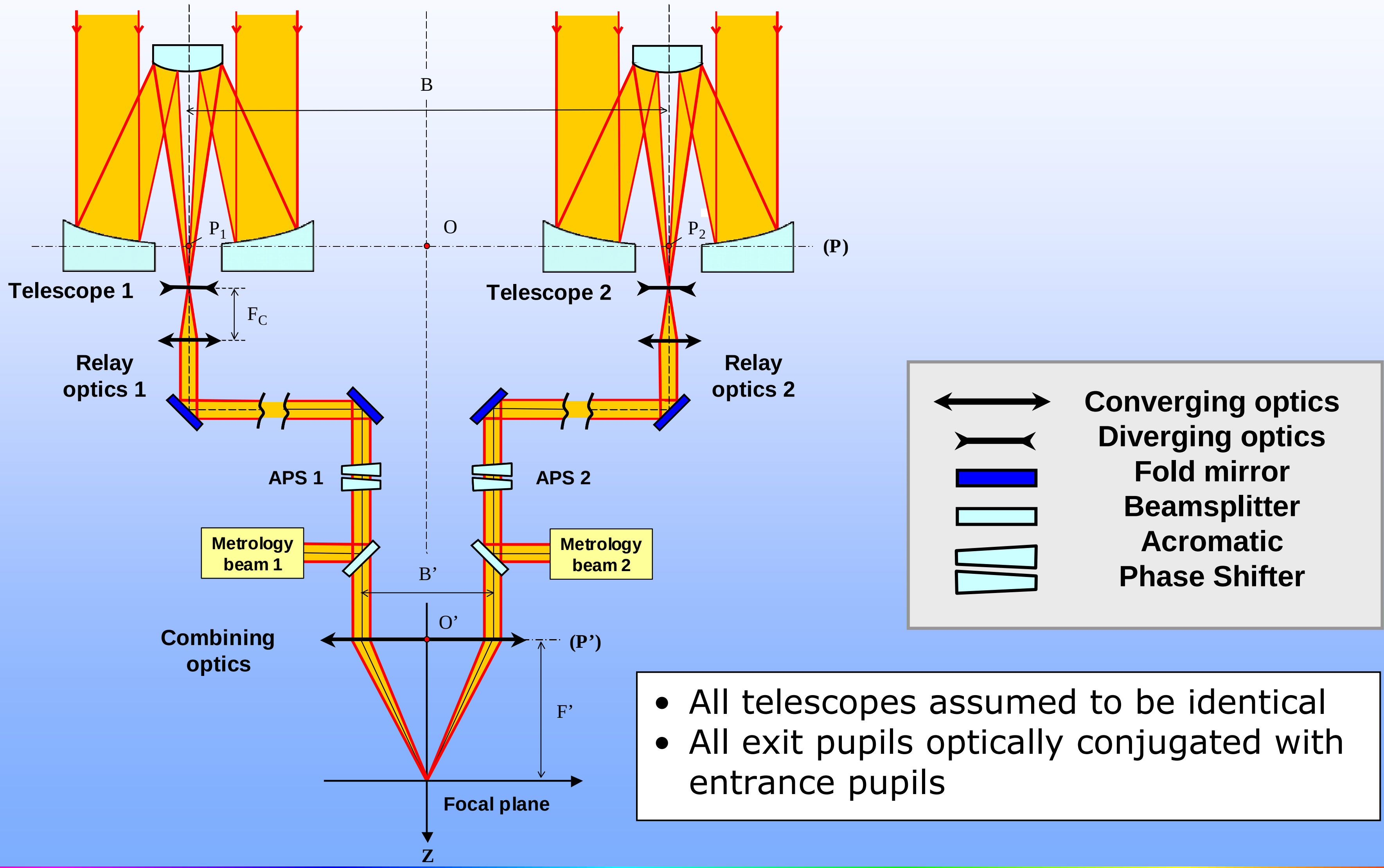
Imaging and nulling properties of sparse-aperture Fizeau interferometers

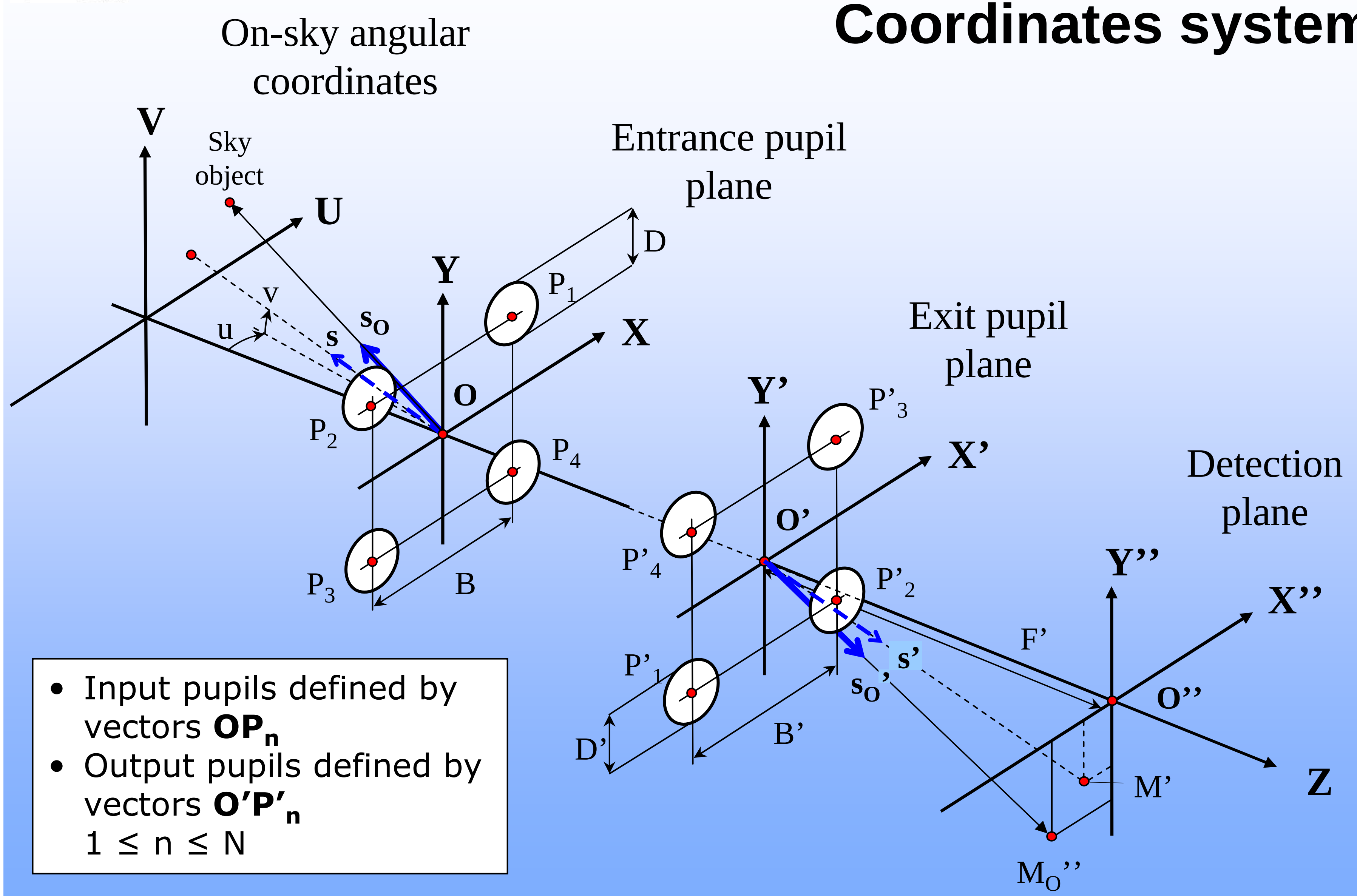
François Hénault
Institut de Planétologie et d'Astrophysique de Grenoble,
Université Joseph Fourier, CNRS, B.P. 53, 38041 Grenoble – France

General layout of a Fizeau interferometer



- All telescopes assumed to be identical
- All exit pupils optically conjugated with entrance pupils

Coordinates systems



- Input pupils defined by vectors $\mathbf{OP_n}$
- Output pupils defined by vectors $\mathbf{O'P'_n}$

  $1 \le n \le N$

Image of a sky object projected onto the sky

Most general Object-Image relationship :

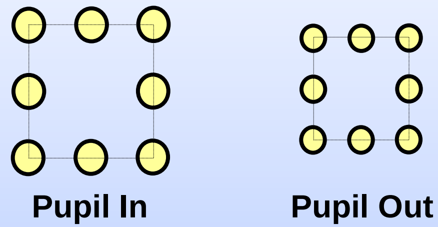
$$I(\mathbf{s}) = \iint_{\mathbf{s_O} \in \Omega_O} O(\mathbf{s_O})\, PSF_T(\mathbf{s}-\mathbf{s_O}) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[ik\,\xi(\mathbf{s_O},\mathbf{s})] \right|^2 d\Omega_O$$

with $\xi(\mathbf{s_O},\mathbf{s}) = \mathbf{s_O}\,\mathbf{OP_n} - \mathbf{s}\,\mathbf{O'P'_n} / m$

- $PSF_T(\mathbf{s})$: PSF of an ***individual*** collecting telescope, projected back onto the sky
- a_n : amplitude transmission factor of the n^{th} telescope
- φ_n : phase-shift along the n^{th} interferometer arm for cophasing or nulling purpose
- $k = 2\pi/\lambda$: wavenumber of ***monochromatic*** electro-magnetic field
- m : optical compression factor between telescopes and their relay optics

Golden rule of Fizeau interferometers

- First-order approximation
- Necessary condition $\mathbf{O'P'_n} = m\,\mathbf{OP_n}$



- Object-Image relationship

$$I(\mathbf{s}) = \left[\mathrm{PSF_T}(\mathbf{s})\ F(\mathbf{s})\right] * O(\mathbf{s})$$

- Invariant PSF over the Field of view (FoV)

$$F(\mathbf{s}) = \mathrm{PSF_T}(\mathbf{s}) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[i\, k\, \mathbf{s}\, \mathbf{OP_n}] \right|^2$$

Examples of application

- Effect of pupil aberrations
- Deviations from the “golden rule” : non-homothetic exit pupil combiners
- Simulation of images in nulling mode

Effect of pupil aberrations

- Develop all vectors at 2nd-order, e.g.

$$\mathbf{s} = \begin{cases} \sin u \approx u \\ \cos u \sin v \approx v \\ \cos u \cos v \approx 1 - u^2/2 - v^2/2 \end{cases} \qquad \mathbf{O'P_n'} = \begin{cases} x_n'\left(1 + dz_n'/F'\right) \\ y_n'\left(1 + dz_n'/F'\right) \\ dz_n' \end{cases}$$

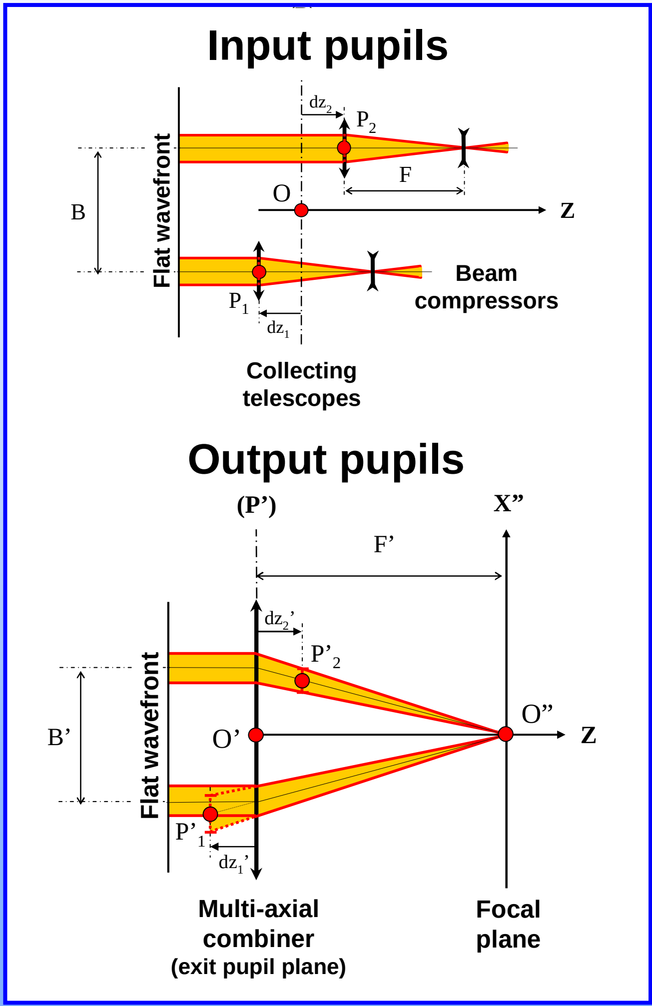
- Insert into the OPD :

$$\xi(\mathbf{s_O},\mathbf{s}) \approx dz_n - dz_n'/m + u_0 x_n + v_0 y_n - \left(u\, x_n' + v\, y_n'\right)\left(1 + dz_n'/F'\right)/m - dz_n\left(u_0^2 + v_0^2\right)/2 - dz_n'\left(u^2 + v^2\right)/2m$$

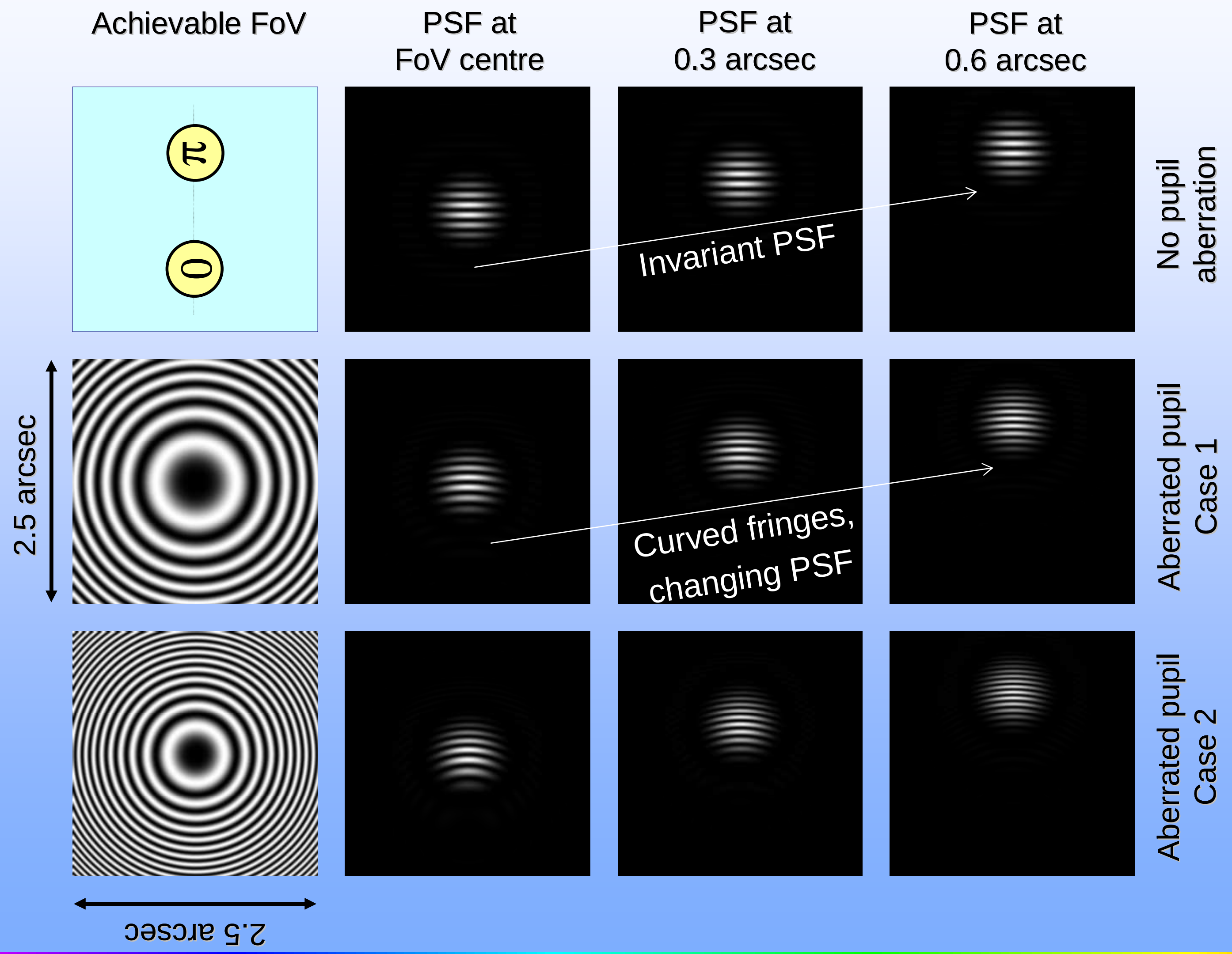
- Use general Object-Image relation :

$$I(\mathbf{s}) = \iint_{\mathbf{s_O} \in \Omega_O} O(\mathbf{s_O})\, PSF_T(\mathbf{s} - \mathbf{s_O}) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[ik\,\xi(\mathbf{s_O},\mathbf{s})] \right|^2 d\Omega_O$$

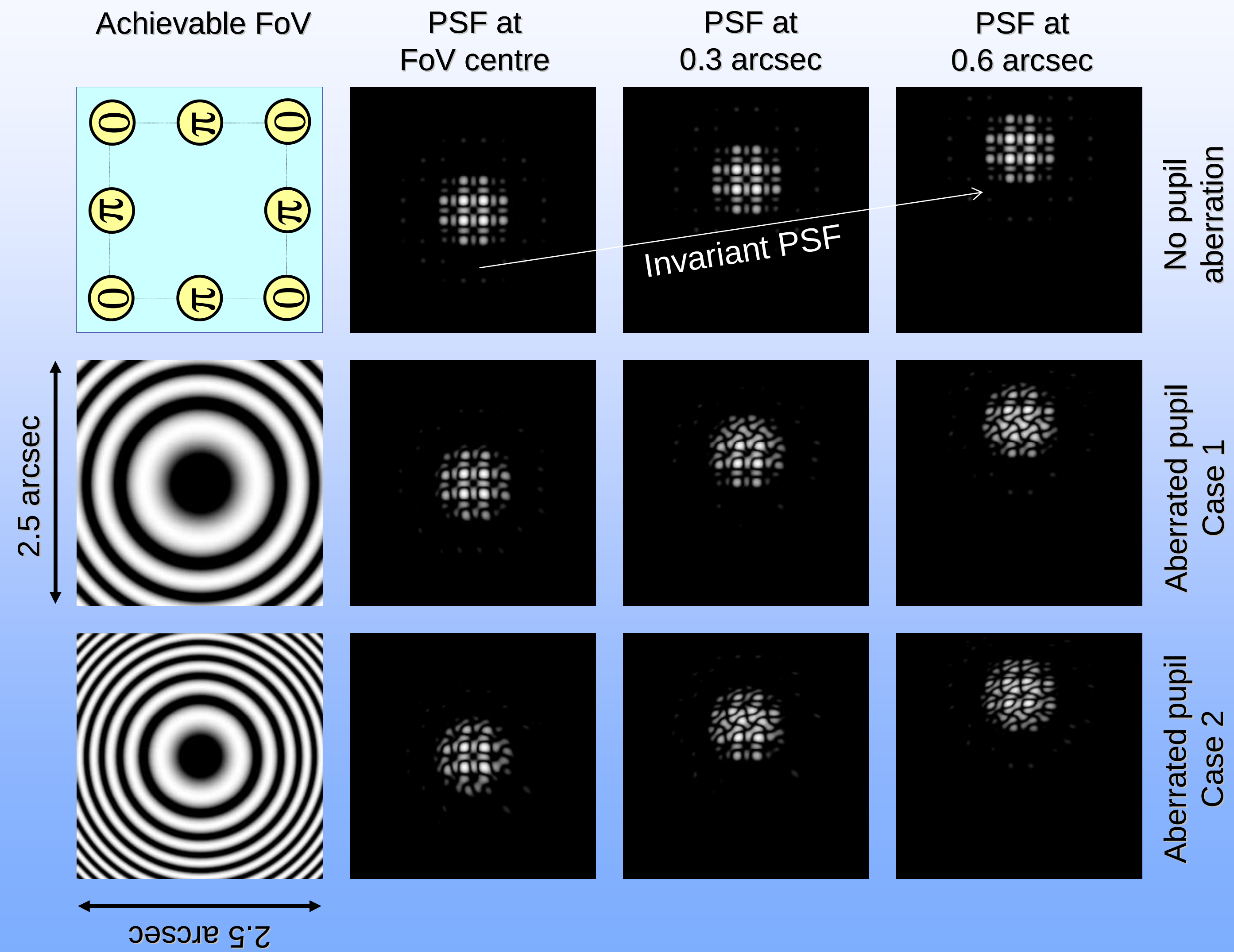
- No longer a convolution operator !



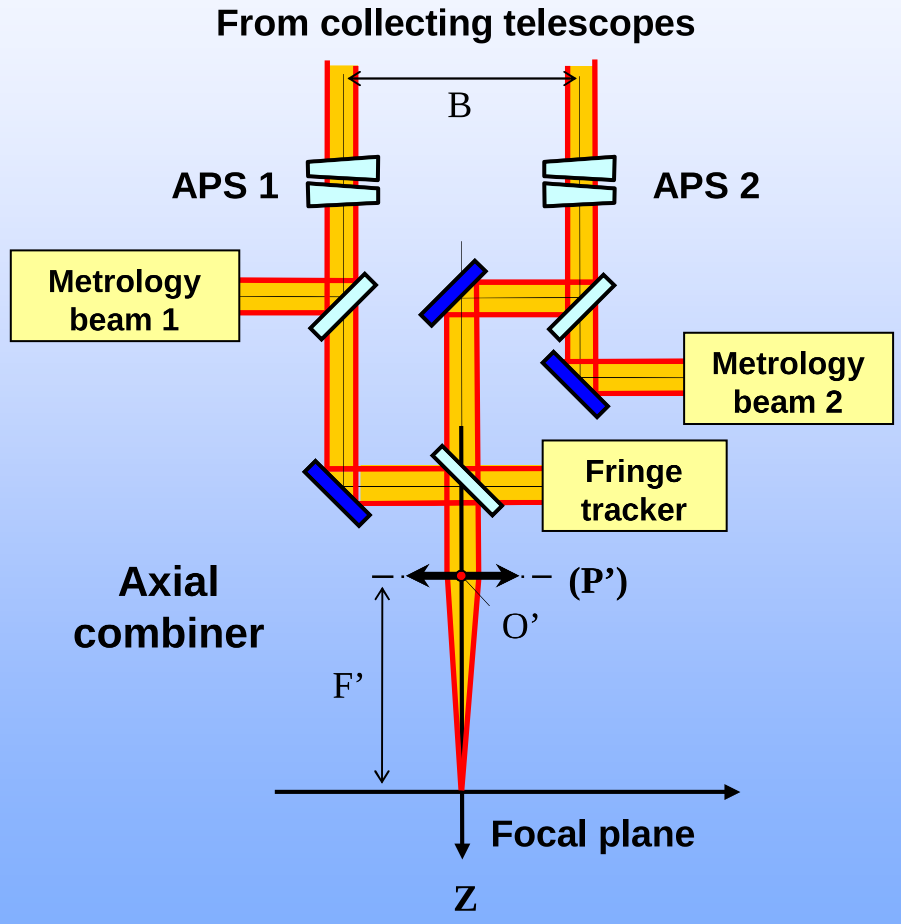
Pupil aberration: 2 telescopes

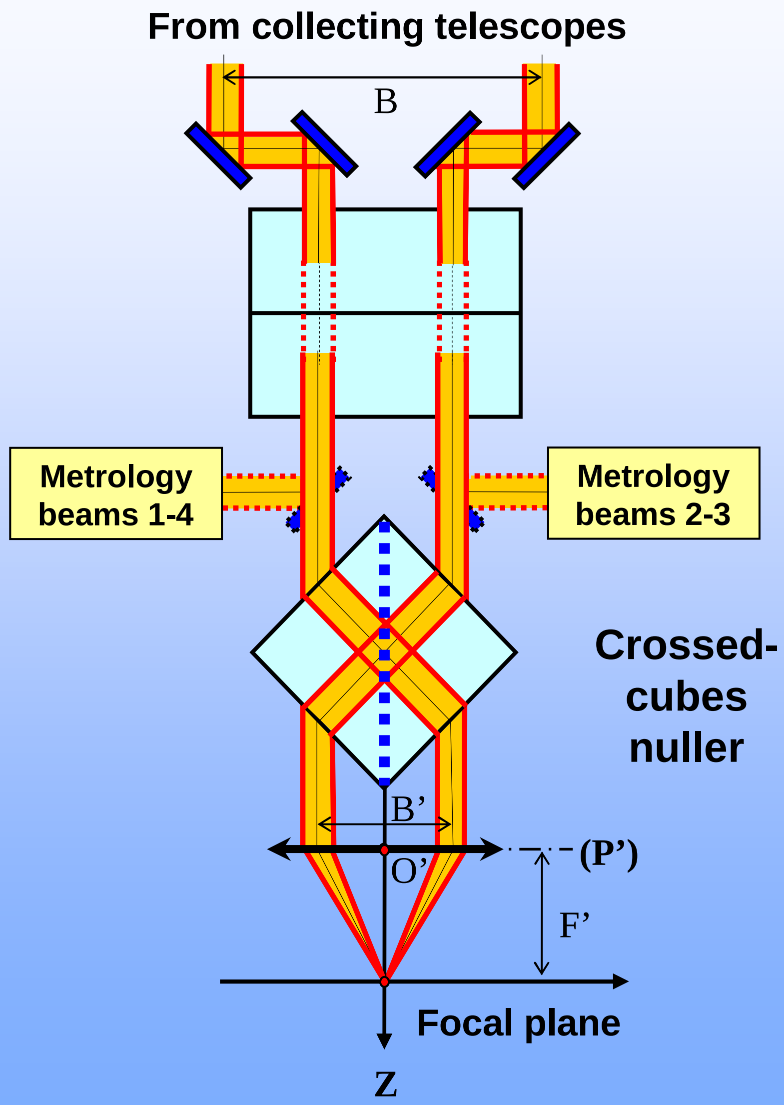


Pupil aberration: 8 telescopes

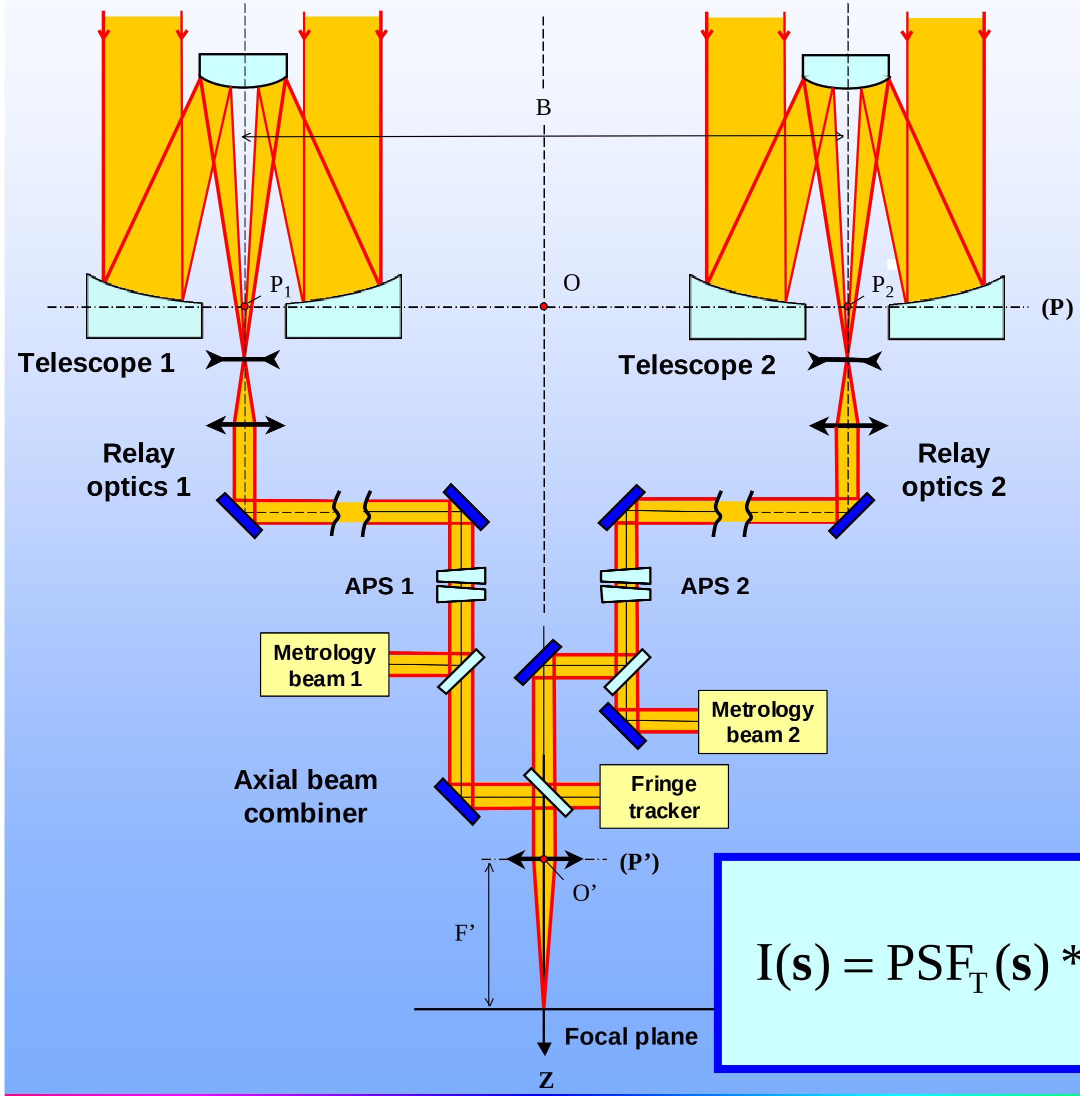


Different combining schemes





Axially Combined Interferometer (ACI)

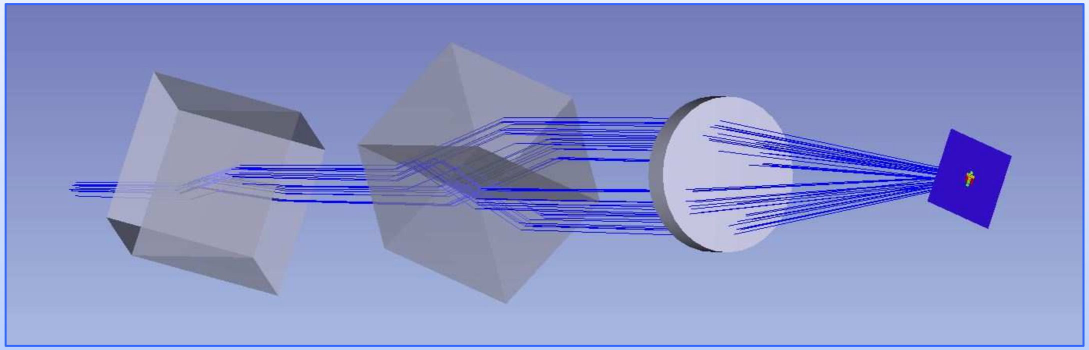


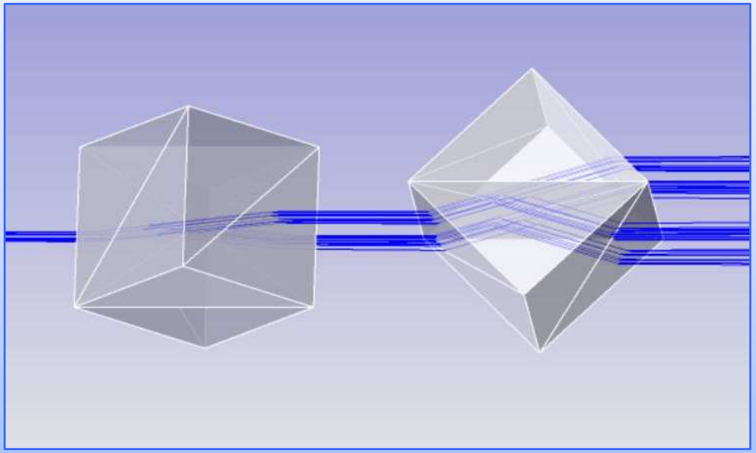
- Co-axial combination by means of a balanced set of beamsplitters
- Extinguishes all diffracted light originating from central star → Only leaves planet photons
- Specific Object-Image relationship

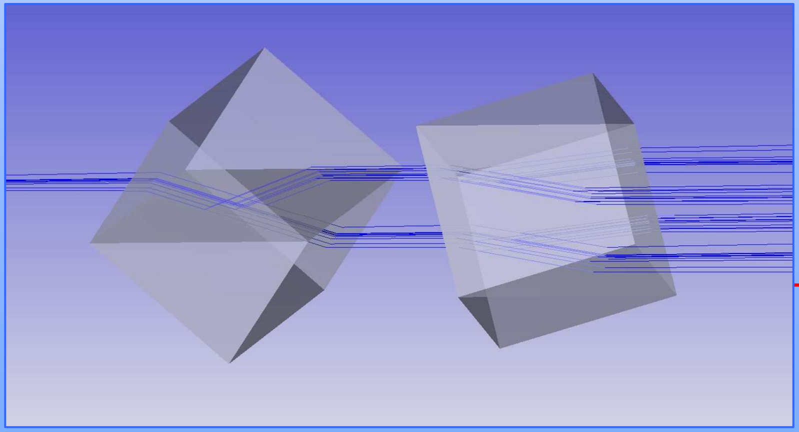
$$I(\mathbf{s}) = PSF_T(\mathbf{s}) * \left\{ \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[ik\,\mathbf{s}\,\mathbf{OP_n}] \right|^2 O(\mathbf{s}) \right\}$$

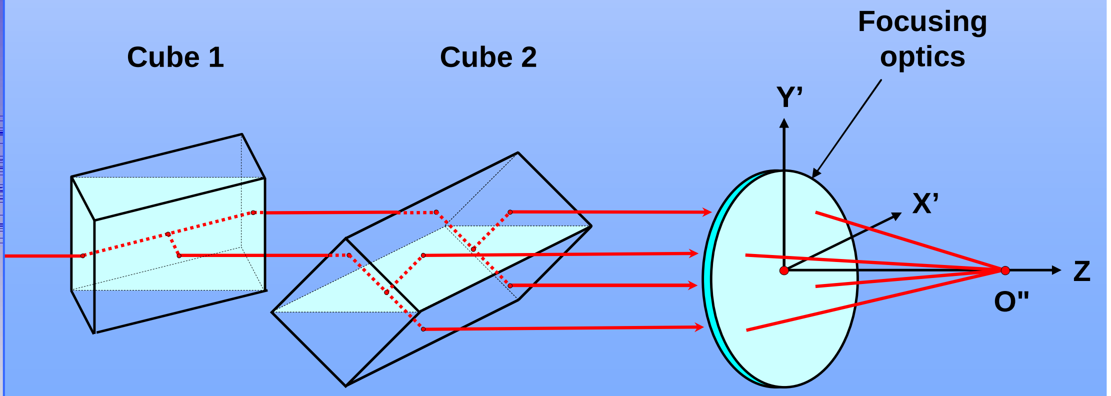
Crossed-cubes nuller (CCN)

- See the "Cheapest nuller in the World":
  - One talk on Thursday afternoon, 2 posters on Wednesday afternoon

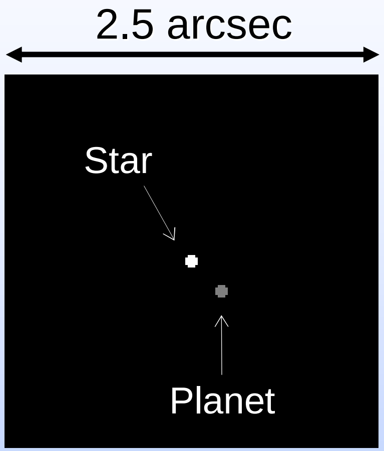




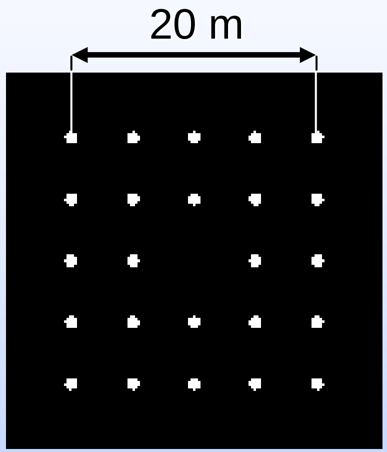




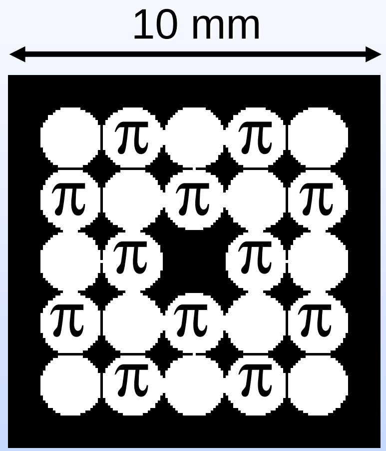
Pseudo-images of a companion



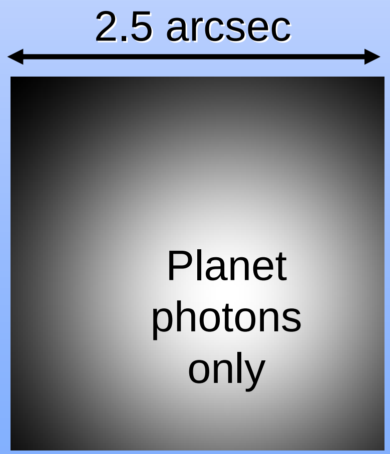
Fake sky objects



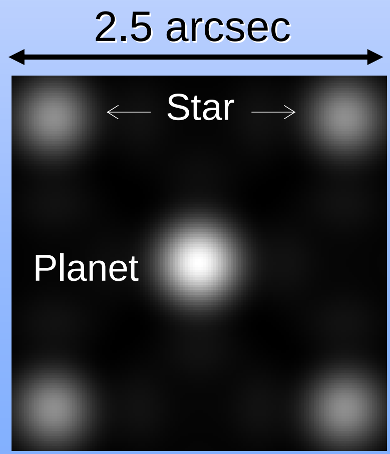
Telescope array



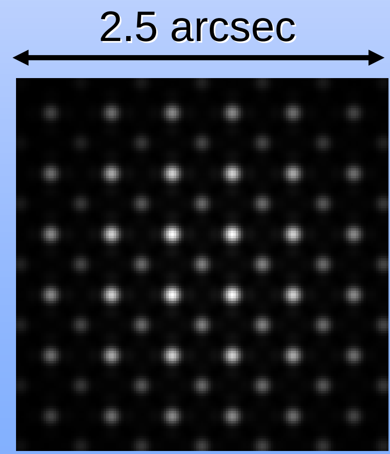
Phase-shifts maps



Axial combiner



Maximal densification



Fizeau interferometer

Conclusion

- A simple Fourier optics formalism allows fast evaluation of nulling Fizeau interferometers performance
  - Including PSF, achievable Field of view (FoV), nulling maps and pseudo-images
  - Whatever is the telescope number N
  - In presence of optical defects (pupil aberrations)
  - Also applicable to non-Fizeau interferometers
- Simple formalism, no actual Fourier or Fresnel transforms required (gain in accuracy and computing time)
- Simulations show that that the most efficient nulling schemes should use axial combiners or multi-axial combiners with maximal densification (e.g. Crossed-cubes nuller)

Previous publications

- "Fibered nulling telescope for extra-solar coronagraphy," Optics Letters vol. 34, n° 7, p. 1096-1098 (2009)
- "Computing extinction maps of star nulling interferometers," Optics Express vol. 16, 4537-4546 (2008)
- "Fine art of computing nulling interferometer maps," Proceedings of the SPIE 7013, n°70131X (2008)
- "Simple Fourier optics formalism for high angular resolution systems and nulling interferometry," JOSA A vol. 27, p. 435-449 (2010)
- "PSF and field of view characteristics of imaging and nulling interferometers," Proceedings of the SPIE vol. 7734, n°773419 (2010)
- "Imaging power of multi-fibered nulling telescopes for extra-solar planet characterization," Proceedings of the SPIE vol. 8151, n°81510A (2011)
- "Cheapest nuller in the world: crossed beamsplitter cubes," Proceedings of the SPIE vol. 9146, this conference

Perhaps one synthesis paper some day…