PSF and Field of View characteristics of imaging and nulling interferometers

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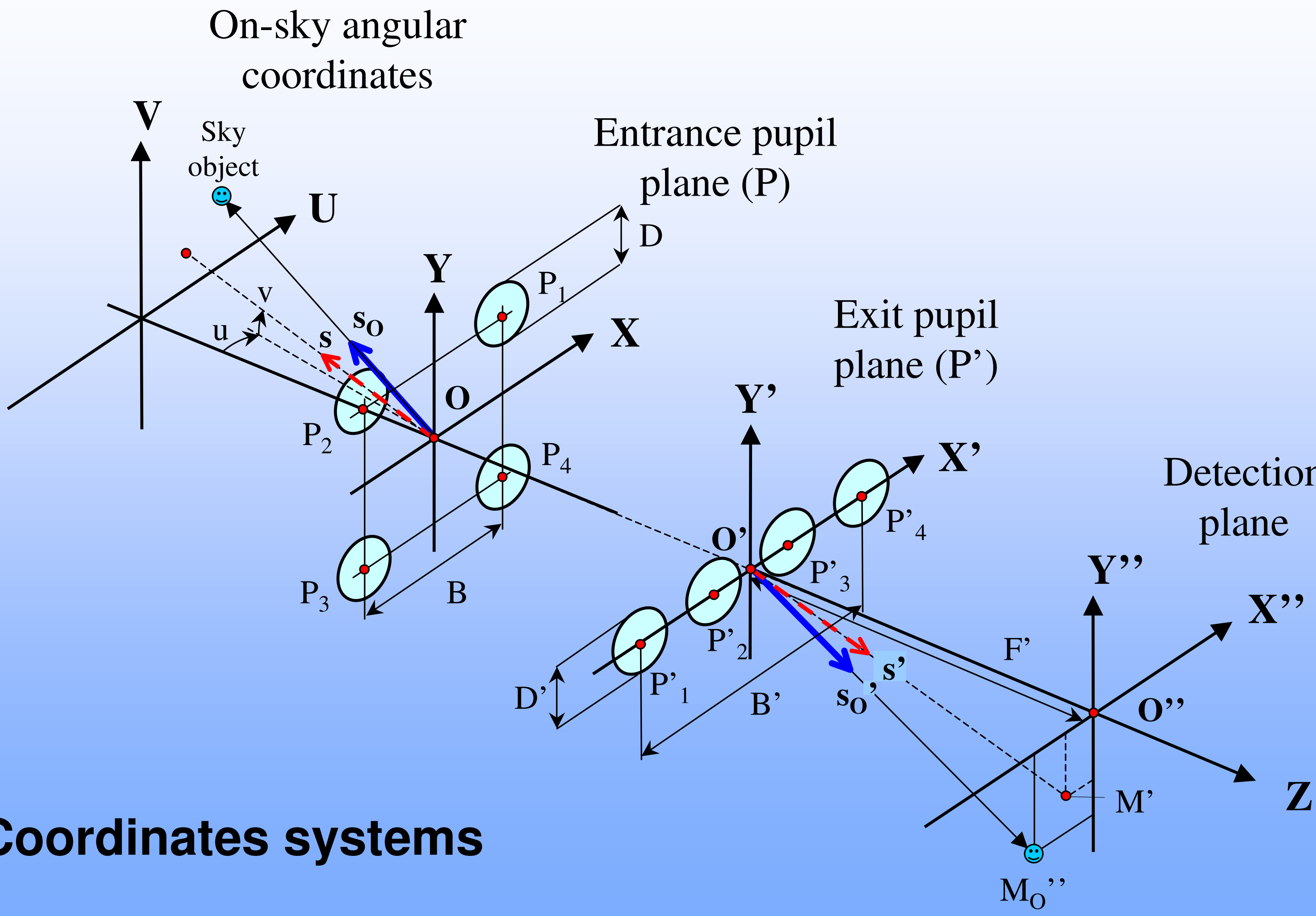
Opening questions

In the frame of Darwin/TPF-I exoplanet finding space missions…

- Do nulling maps depend on the type of combining optics (axial vs. multi-axial combination schemes) ?
- Has it some consequence on their "nulling imaging" capacity ?

Previous publications

- "Simple Fourier optics formalism for high angular resolution systems and nulling interferometry," JOSA A **27**, p. 435-449 (2010)
- "Fibered nulling telescope for extra-solar coronagraphy," Optics Letters **34**, n° 7, p. 1096–1098 (2009)
- "Computing extinction maps of star nulling interferometers," Optics Express **16**, 4537-4546 (2008)



Coordinates systems

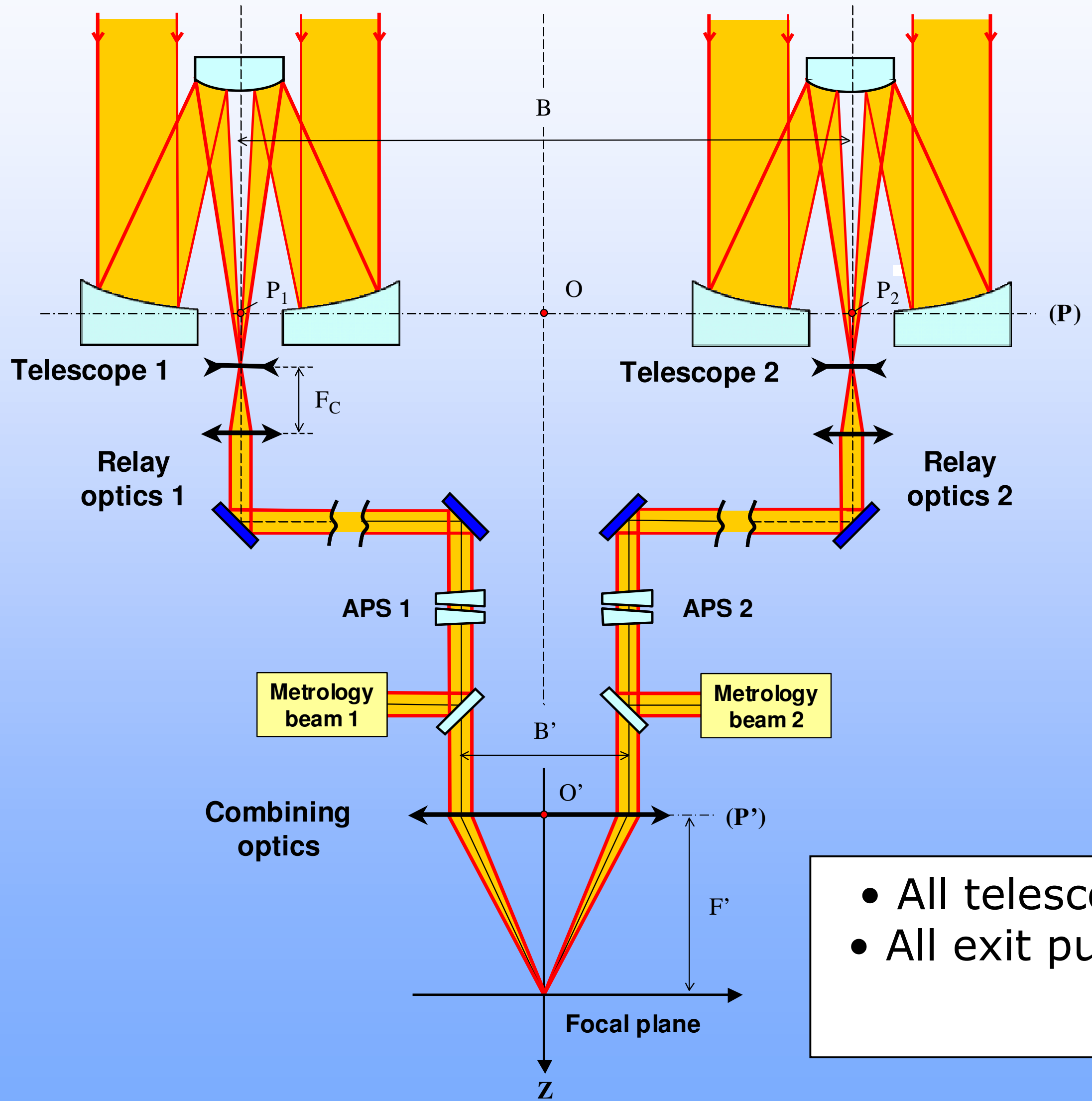
Image of a sky object O(s) projected back on sky

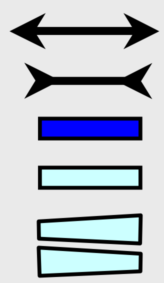
$$\mathrm{I}(\mathbf{s}) = \iint_{\mathbf{s_O} \in \Omega_O} \mathrm{O}(\mathbf{s_O})\, \mathrm{PSF_T}(\mathbf{s} - \mathbf{s_O}) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp\left[ik\left(\mathbf{s_O}\,\mathbf{OP_n} - \mathbf{s}\,\mathbf{O'P'_n}/m\right)\right] \right|^2 d\Omega_O$$

with

- $\mathrm{PSF_T}(\mathbf{s})$: PSF of one ***individual*** collecting telescope, being projected back on-sky
- a_n : amplitude transmission factor of the n^{th} telescope
- φ_n : phase-shift along the n^{th} interferometer arm for cophasing or nulling purpose
- $k = 2\pi/\lambda$: wavenumber of ***monochromatic*** electro-magnetic field
- *m* : optical compression factor between telescopes and their relay optics

General layout of a multi-aperture interferometer





- All telescopes assumed to be identical
- All exit pupils optically conjugated with entrance pupils

PSF and Field of View characteristics

- Generalized Point Spread Function (PSF)

$$\mathrm{PSF_G}(\mathbf{s},\mathbf{s_O}) = \mathrm{PSF_T}(\mathbf{s}) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[-ik\,\mathbf{s}\,\mathbf{O'P'_n}/m] \exp[ik\,\mathbf{s_O}\,(\mathbf{OP_n} - \mathbf{O'P'_n}/m)] \right|^2$$

  - Changing over the whole instrument Field of View
  - Object-Image relationship is ***not*** a convolution product ☹
- Maximal achievable Field of View (FoV)
  - Neglecting any kind of apertures or stops,
  - Neglecting geometrical aberrations and diffraction effects

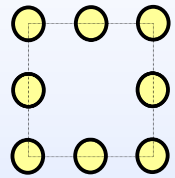
$$\mathrm{FoV}(\mathbf{s}) = \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[ik\,\mathbf{s}\,(\mathbf{OP_n} - \mathbf{O'P'_n}/m)] \right|^2$$

  - Suitable for fast polychromatic FoV computations:

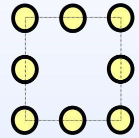
$$\mathrm{FoV}_{\delta\lambda}(\mathbf{s}) = \int_{\delta\lambda} \mathrm{FoV}_\lambda(\mathbf{s})\, B_{\delta\lambda}(\lambda)\, d\lambda \Big/ \int_{\delta\lambda} B_{\delta\lambda}(\lambda)\, d\lambda$$

Golden rule of Fizeau interferometers

- Only occurs when: $\boxed{\mathbf{O'P'_n} = m\,\mathbf{OP_n}}$
- Generalized PSF becomes:



Pupil In



Pupil Out

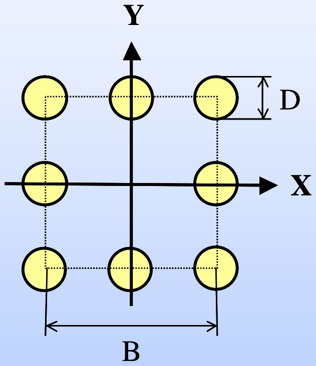
$$\mathrm{PSF_G}(\mathbf{s},\cancel{\mathbf{s}_0}) = \mathrm{PSF_T}(\mathbf{s}) \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[-ik\,\mathbf{s}\,\mathbf{O'P'_n}/m] \right|^2$$

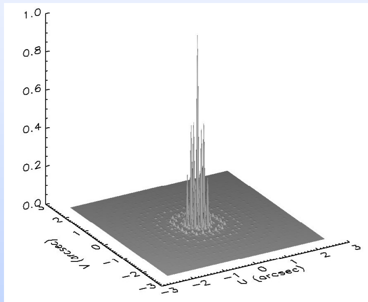
  - Constant over the whole FoV
  - Classical Object-Image relationship I(**s**) = O(**s**) * PSF(**s**) holds ☺
- Maximal achievable Field of View:
  - Becomes infinite whatever the wavelength
  - Constant transmission equal to:

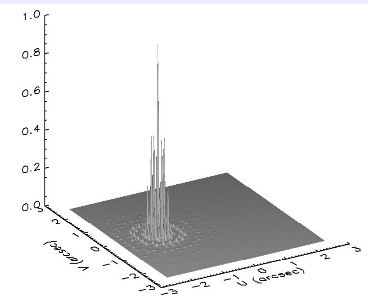
$$\mathrm{FoV}(\cancel{\mathbf{s}}) = \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \right|^2$$

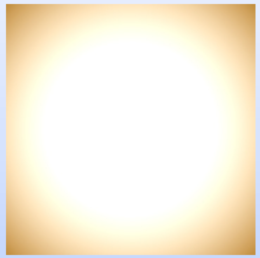
Numerical simulations: a 8-telescope Fizeau interferometer in imaging and nulling modes

Input pupils	PSF (FoV center)	PSF (half FoV)	Maximal achievable FoV

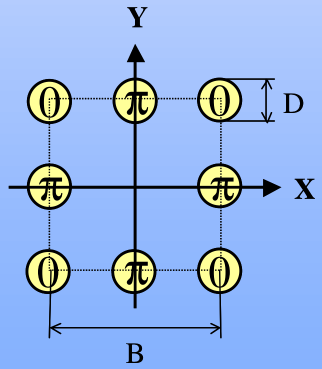


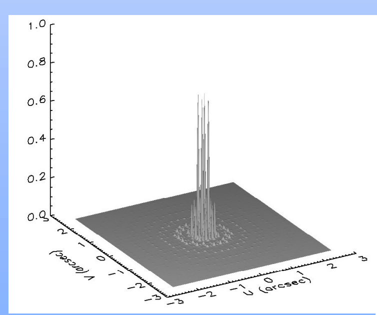


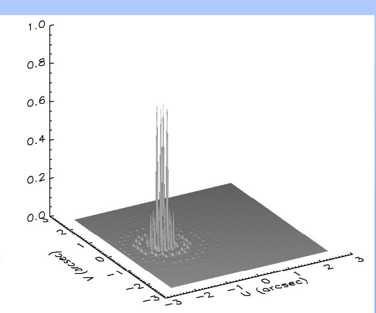


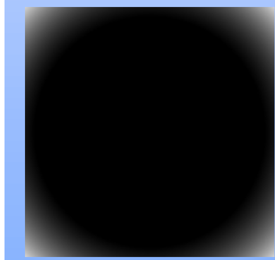


5 arcsec





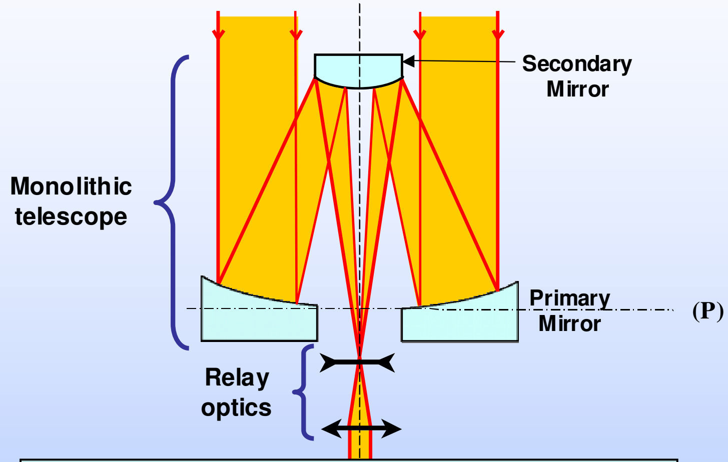




5 arcsec

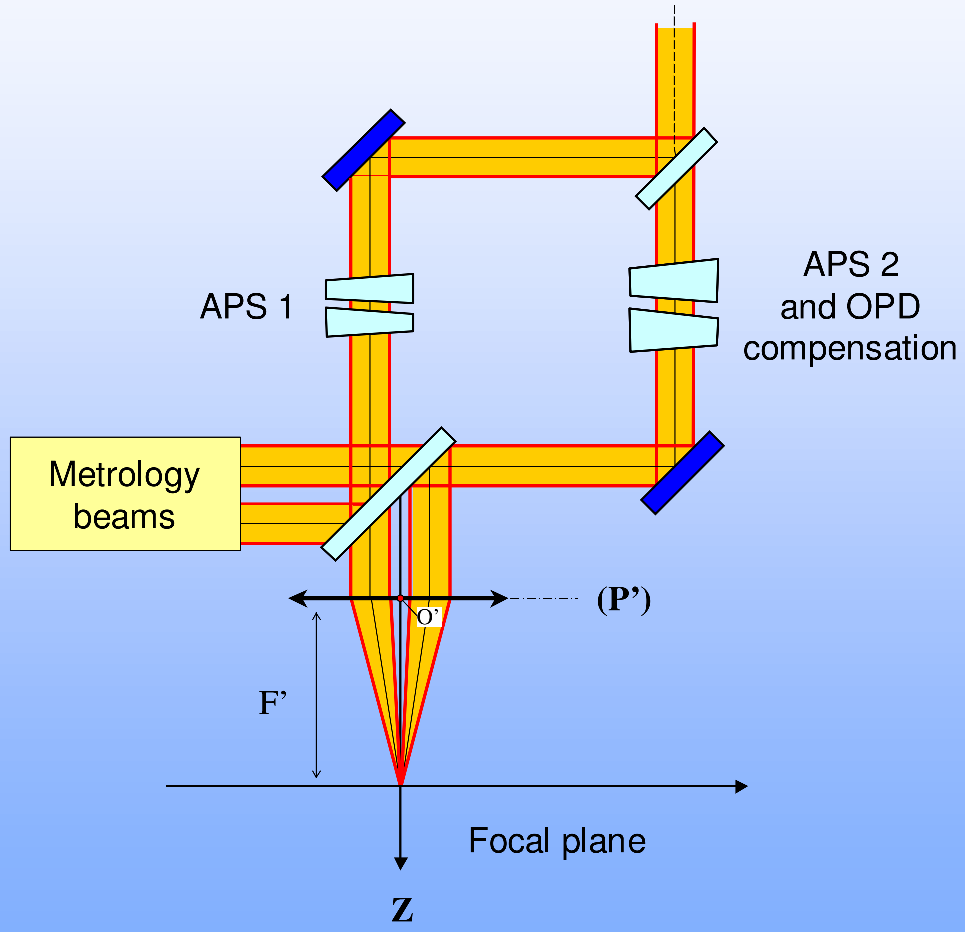
- Golden rule extends destructive fringe over the whole FoV, killing the central star ***and*** all its surrounding planets (this is not what we want ☹)

Sheared-Pupil Telescopes (SPT)



May be replaced with a modified Mach-Zehnder combiner

[or Michelson equipped with cube-corners]

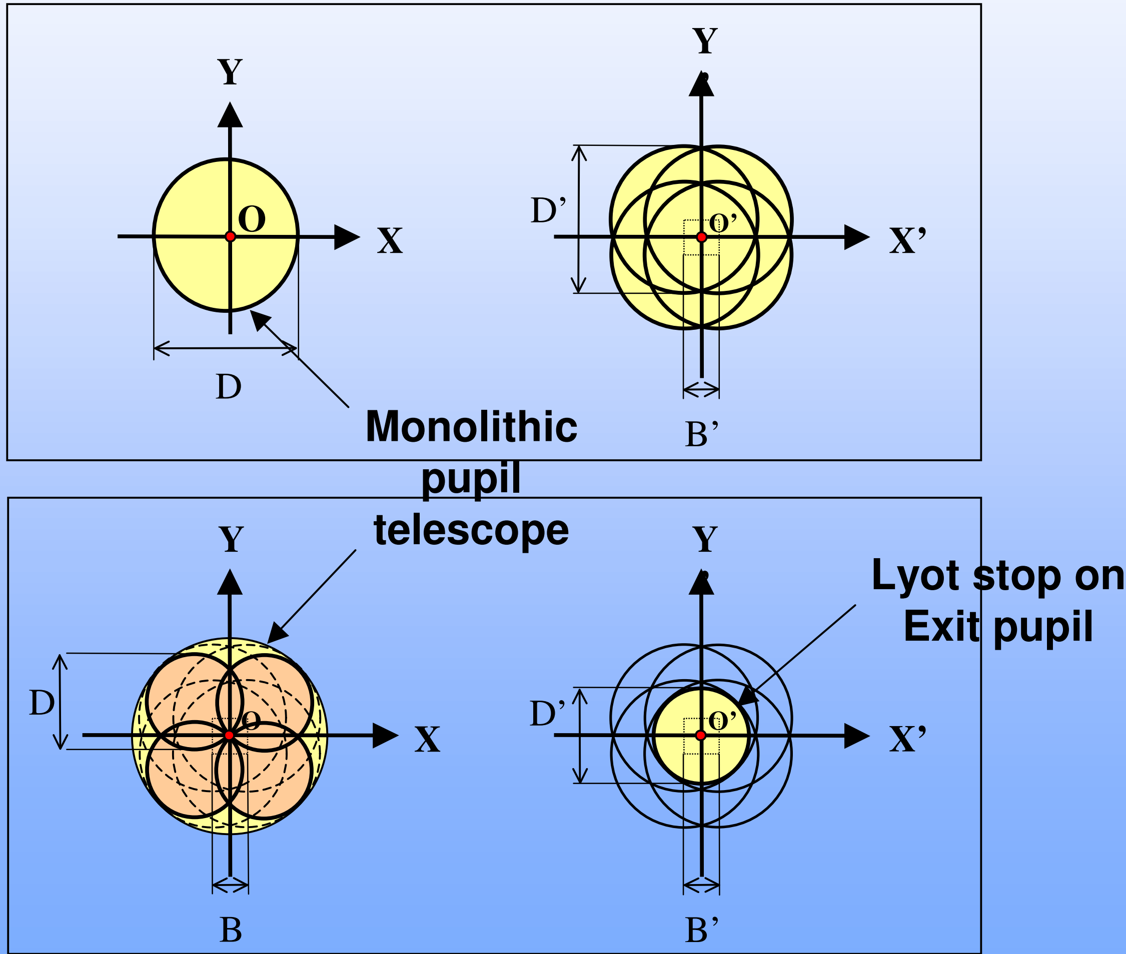


Two different types of Sheared-Pupil Telescope

Input pupil plane Output pupil plane

Unmasked output sub-pupils

Masked output sub-pupils



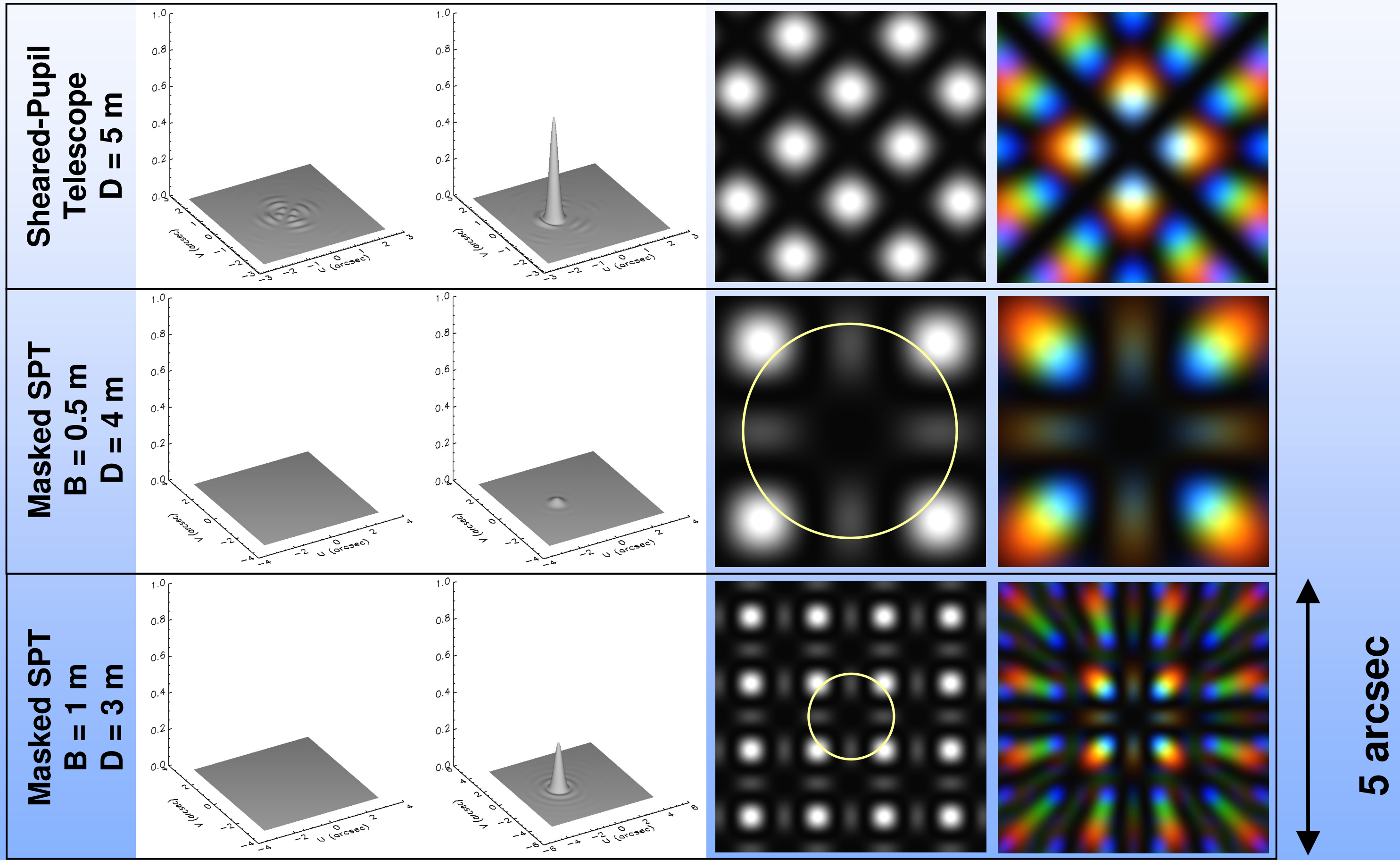
Interest and limitations of nulling SPTs

- Usable for exploratory science missions: exo-zodi characterization, Jupiter-like planets…
- If rotating, allow to validate most of the Darwin/TPF-I algorithms envisaged for planets finding and characterization
- When unmasked, they concentrate energy in very small core, overcoming Rayleigh's diffraction limit ☺
- But no real super-resolving power ☹, since PSF are sharpened ***after*** sub-aperture filtering:

Specific Object-Image relationship:

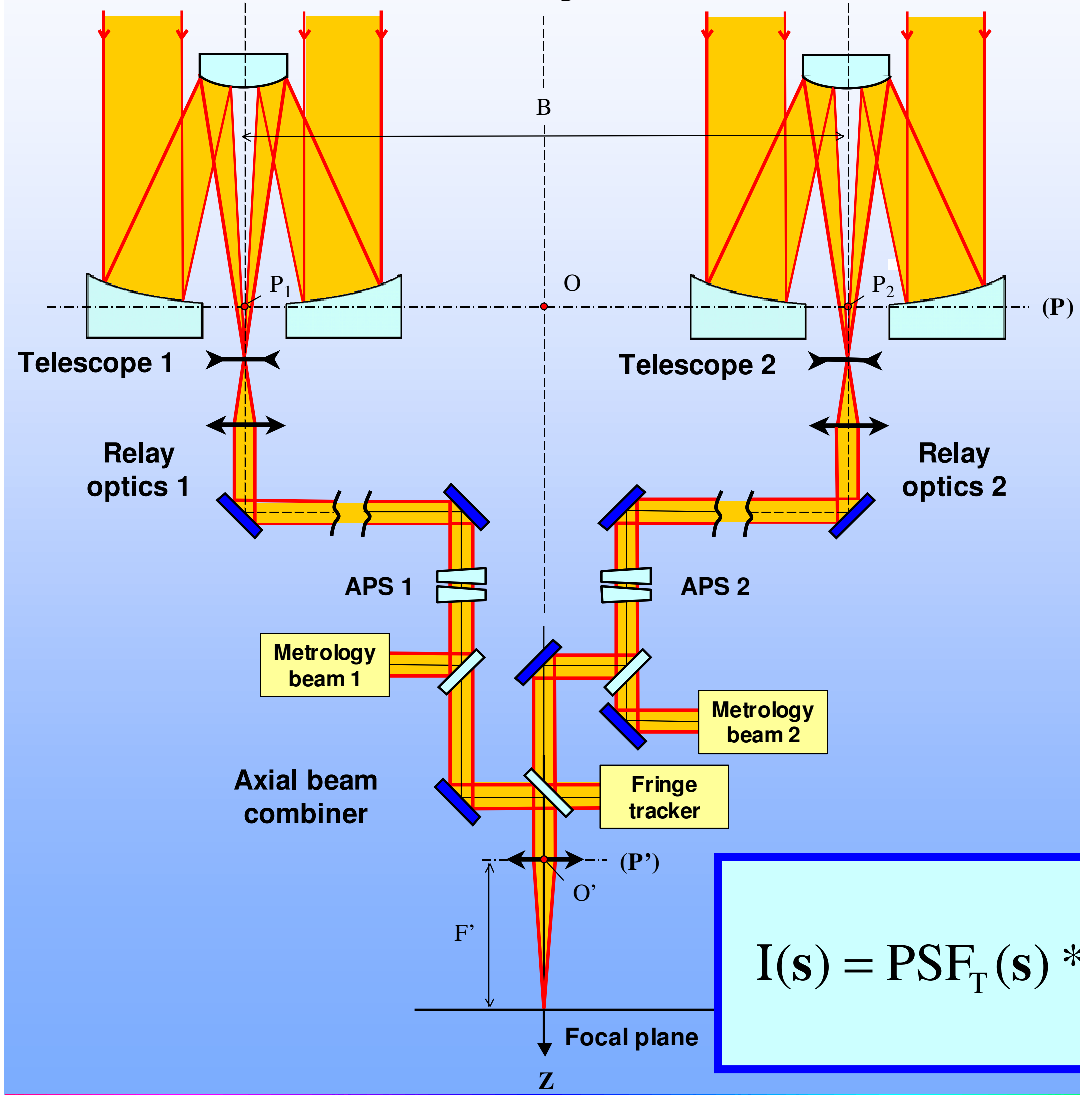
$$\mathrm{I}(\mathbf{s}) = \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[-ik\,\mathbf{s}\,\mathbf{O'P'_n}/m] \right|^2 \times [\mathrm{PSF_T}(\mathbf{s}) * \mathrm{O}(\mathbf{s})]$$

PSF and Field of View simulations of SPTs



- Unmasked SPT: high throughput, residual star leakage
- Masked SPT: no leakage, enlarged nulled area of low throughput

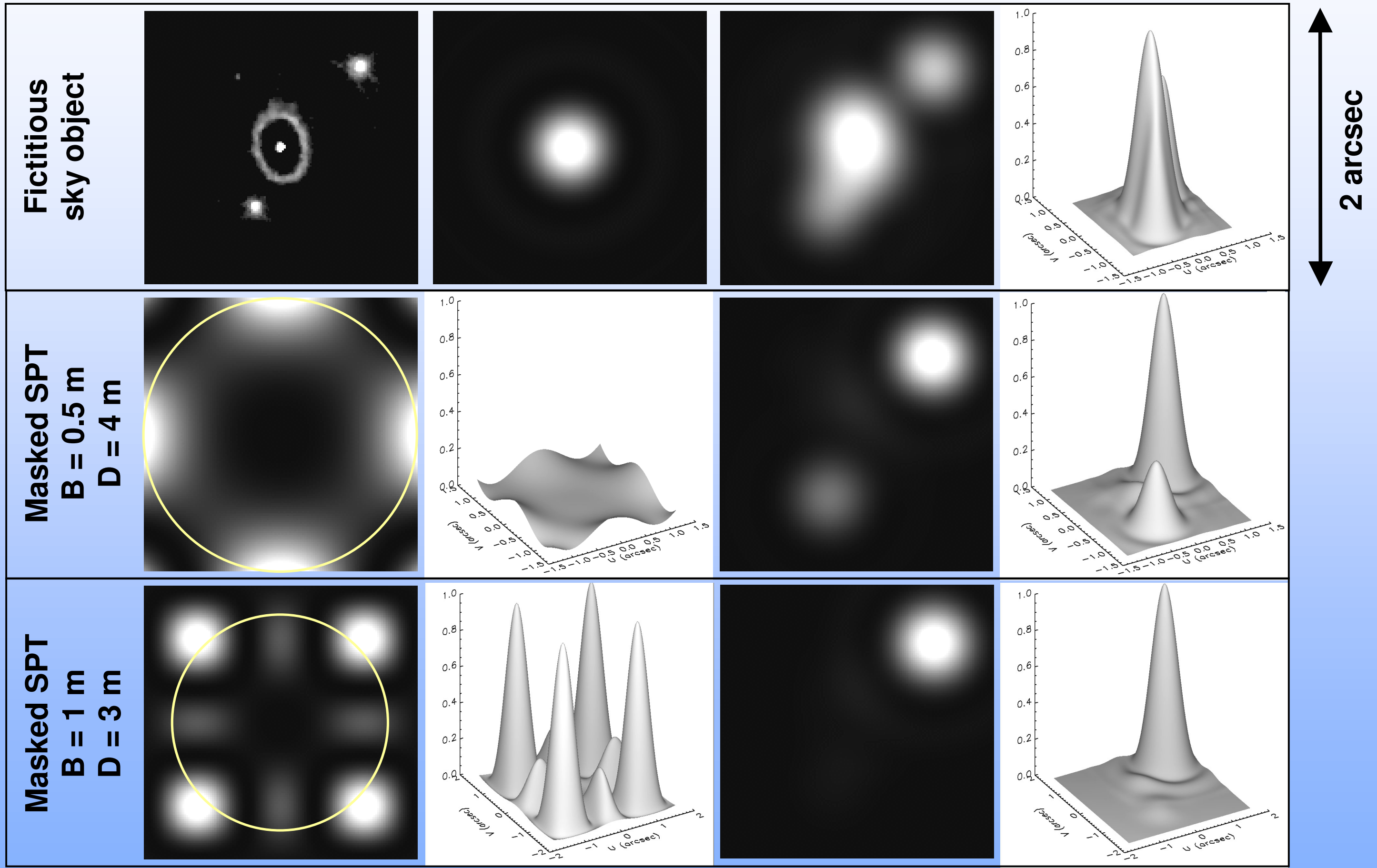
Axially Combined Interferometer (ACI)



- Co-axial recombination by means of a balanced set of beamsplitters
- Equivalent to the previous monolithic, masked SPT
- Nulls all diffracted light originating from central star
- Specific Object-Image relationship:

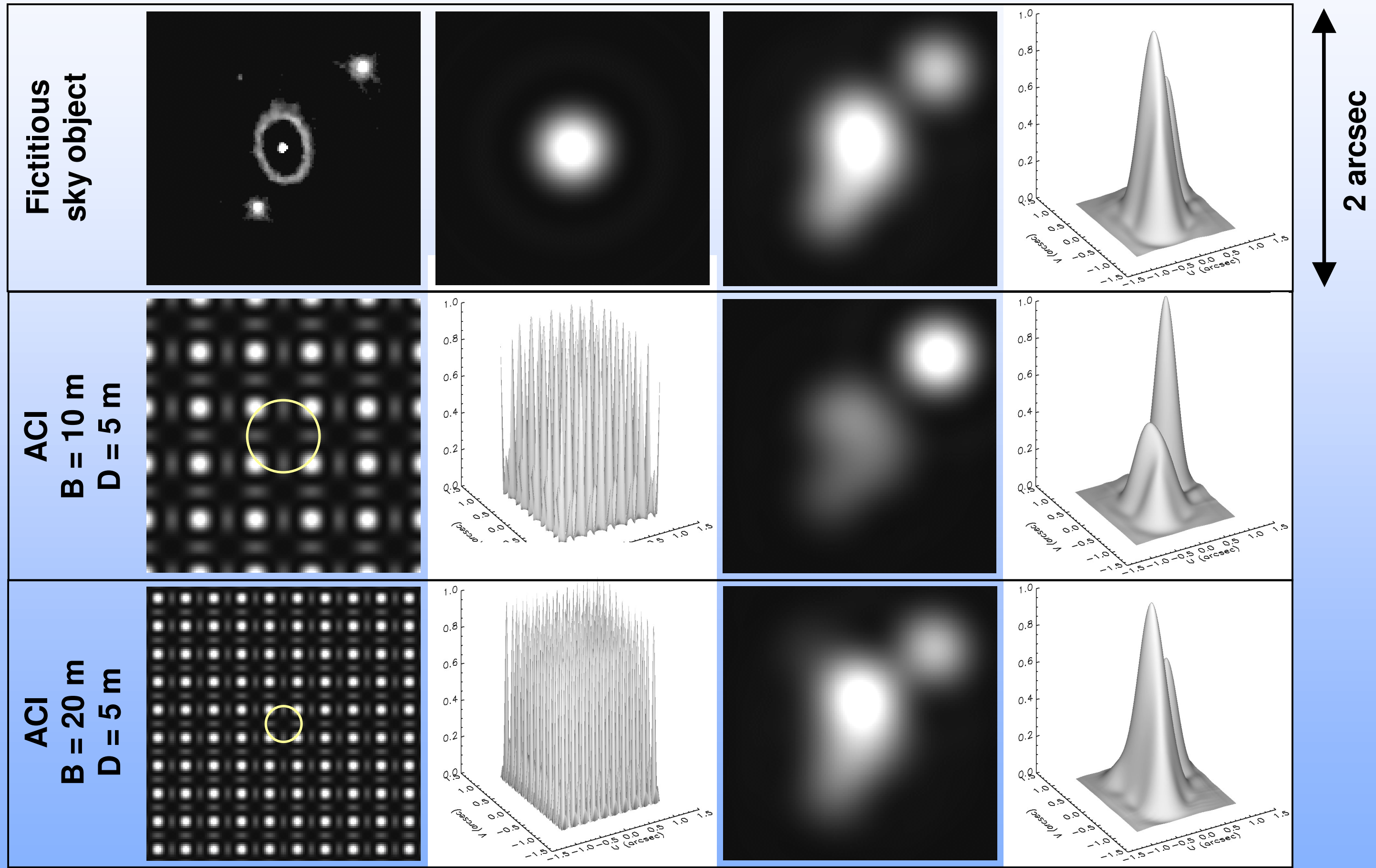
$$I(\mathbf{s}) = PSF_T(\mathbf{s}) * \left\{ \left| \sum_{n=1}^{N} a_n \exp[i\varphi_n] \exp[ik\,\mathbf{s}\,\mathbf{OP_n}] \right|^2 O(\mathbf{s}) \right\}$$

Nulling imaging capacities of SPT



- **No gain in angular resolution, but diffracted starlight cleaned *before* final image blurring**
- **Progressive leakage from central objects (to be traded against throughput)**

Nulling imaging capacities of ACI



- For longer baselines, nulling ACI behaves as a single-dish telescope
  → Nulling capacity seems to be lost ☹

Conclusions

- **Nulling maps and nulled images produced by different types of multi-aperture optical systems can be rapidly evaluated by means of a simple Fourier optics formalism**
- **From the results of theory and first numerical computations:**
  - "Golden rule for interferometric imaging" extends the destructive fringe pattern of nulling interferometers over their whole Field of View
  - A nulling monolithic, sheared-pupil telescope is an attractive solution → Requires further tradeoff on throughput/leakage
  - Theoretical Object-Image relationship of the Bracewell interferometer allows full extinction of diffracted starlight, but no super-resolution is possible

[Please retain: this is all very preliminary, eventually anecdotal and somewhat heuristic. Further work is required and cooperations are welcome]